Cyclic Logic Structure (CLS)

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The Cyclic Logic Structure (CLS) partitions the positive integers into self-similar, non-overlapping rings (halos) and defines sparse query operations into the primes, yielding a modular framework for state generation and alignment. Below is its core mathematical specification, derived directly from the recursive definitions. CLS is a deterministic oracle-based combinator for sparse, cyclic logic; extensible to hierarchical models but rigorously defined for finite depths.

1 Halo Partitioning

The positive integers $\mathbb{Z}^+ = 1, 2, 3, \ldots$ are partitioned into halos $H_h = [s_h, e_h] for h \in \mathbb{N}$ with s_h inclusive start, e_h inclusive end, and length $L_h = e_h - s_h + 1$.

Recursive lengths:$L_1 = 20$;$L_h = 3L_{h-1}$ for$h \geq 2$, so $L_h = 20 \times 3^{h-1}$.

Boundaries: $s_1 = 1$, $e_1 = 20$; $s_h = e_{h-1} + 1$, $e_h = s_h + L_h - 1$ for $h \geq 2$.

Membership: For $n \geq 1$, $h = min i | e_i \geq n$, so $n \in H_h$.

Yields complete, gapless tiling: $\mathbb{Z}^+ = \bigsqcup_{h=1}^{\infty} H_h$.

2 Generalization to Arbitrary Channel Count (G)

The choice of base length 20 and multiplier 3 is privileged but not unique. CLS generalizes to any channel count $G \geq 4$ (or $G = 2$ in degenerate cases) by setting $L_1 = mG (m \geq 1)$ and multiplier $r \geq 2$ coprime to G. All core properties (self-similar tiling, exact phase alignment on G-grid, reversible folds, avalanche dynamics, NAND-completeness) are preserved.

3 Phase Assignment

For $n \in H_h$, phase $\phi(n) = (n - s_h)/L_h \times 360^\circ \in [0^\circ, 360^\circ)$.

Cross-halo sharing: $n_1 \in H_{h1}$, $n_2 \in H_{h2}$ share phase iff$(n_1 - s_{h1})/L_{h1} = (n_2 - s_{h2})/L_{h2}$ (fractional equality).

Canonical grid: $gcd(L_h) = 20$ forces phases to align on 20 discrete fractions $0/20, \ldots, 19/20$, yielding 20 radial channels.

4 Modular Reduction

Projection $\pi_h : \mathbb{Z} \to [s_h, e_h]$, $\pi_h(x) = s_h + ((x - s_h) mod L_h)$, where mod yields $[0, L_h - 1]$. Handles negatives correctly (e.g. $\pi_1(-4) = 16$).

5 State Generation

For anchors $n \in A_h$, $k \in \mathbb{N}$, $m+ = \pi_h(n + k)$, $m- = \pi_h(n - k)$, $\sigma_k(n) = (I[m + prime], I[m - prime]) \in 0, 1^2$.

Yields four states: $(1, 1)$ resonant,$(0, 0)$ void, $(1, 0)/(0, 1)$ asymmetric.

6 State Dynamics and Equivalence

Fold-overs: for $k > L_h/4$, reflection $r = L_h - 2(k - L_h/4)$ swaps $(1, 0) \leftrightarrow (0, 1)$; symmetric states $(1, 1)/(0, 0)$ fixed (period-2 oscillation).

Equivalence: $n_1 \sim_k n_2$ iff $\sigma_k(n_1) = \sigma_k(n_2)$; classes form dynamic buses.

Properties: sparsity computable for finite h; universality via 4-state NAND-completeness; full reversibility (folds + k-decrement); no erasure.

7 Multi-Ring Generalization and the Ring Hierarchy

The full power of CLS is revealed when we recognize that the core machinery (self-similar halos of length $L_h = 20 \times 3^{h-1}$, modular projection π_h, the universal probe offset $k \in \mathbb{N}_0$, the 20-fold phase grid, and the 4-state oracle $\sigma_k(n)$) is completely agnostic to how we choose the anchor predicate. Any decidable property that selects a non-empty subset of composites or primes in every sufficiently deep halo defines a new ring. All rings share the identical halo partitioning, identical phase channels, identical k, and identical state-generation rules. Only the density and clustering of anchors differ.

7.1 Core Properties Shared by All Rings

Exact phase alignment on the canonical 20-grid is preserved for any anchor predicate (because every halo length is a multiple of 20).

Fold-over symmetry and perfect reversibility hold unchanged.

4-state NAND-completeness and logical universality are retained.

Cross-ring resonance: anchors on the same phase channel in different rings experience identical k simultaneously, yielding natural hierarchical synchronization.

7.2 The Eight Canonical Rings (Proposed Standard)

In principle there are countably infinitely many decidable predicates, hence infinitely many rings. The eight canonical rings below are proposed as a standard with infinite alternatives. A minimal, universally useful set covering ≈20 orders of magnitude in resonant frequency:

R1 Prime Ring: $is_{prime(n)}$

R2 Semiprime Ring: $\omega(n) = 2$ and square-free

R3 Triprime Ring: $\omega(n) = 3$ and square-free

R4-R6 Mid-prime Rings: $\omega(n) = 4, 5, 6$ square-free

R8+ High-multiplicity: $\Omega(n) \geq 8$ or specific thresholds

RT Twin-Prime Ring: composites $n \geq 2$ lying between twin primes ($n - 1$ and $n + 1$ both prime). Let $A_h = n \in H_h | n\, anchor$

7.3 Hierarchical Implications

Because every ring shares the same k and the same 20 phase channels, slow rings act as immutable parent clocks for billions of fast-ring ticks. Proving a resonant state on a slow ring constitutes automatic proof of enormous work on fast rings (same k was probed billions of times), and the entire hierarchy is implementable in one unified engine; switching rings is a single predicate change.