Physical Measurement as Pure Clock-and-Ruler Comparison

Gabriel Stuckey

November 27, 2025

1 Primitive Operations (GR + QM compatible)

Proper-time interval on a time-like worldline:

$$\Delta\tau = \int \sqrt{-ds^2} \;\rightarrow\; [T]$$

Proper-length interval between two events simultaneous in the local rest frame (Einstein synchronisation with local clock):

$$\Delta\ell = \sqrt{ds^2}\Big|_{\Delta\tau=0} \;\rightarrow\; [L]$$

These are the only two directly measurable, coordinate-independent scalars.

2 Final Base Dimensions

Fix the Compton angular frequency of one standard atom: $\omega_0 \equiv \frac{m_0 c^2}{\hbar} =$ exact (conventional)

Then only two base dimensions remain: $[T]$ and $[L]$.

3 Dimensional Table

Quantity	Traditional	Final
τ	T	T
ℓ	L	L
m, e	M, Q	T^2L^{-1}
$p = mv\gamma$	MLT^{-1}	T
$E = mc^2$	ML^2T^{-2}	L
$\hbar$	ML^2T^{-1}	LT
F (non-gravitational)	MLT^{-2}	1
α = proper acceleration	LT^{-2}	LT^{-2}

4 Proof that $[F] = 1$

Four-acceleration: $A^\mu = du^\mu/d\tau,\ A^\mu u_\mu = 0$

Proper acceleration $\alpha = \sqrt{A^\mu A_\mu}$ = measured with accelerometers $\alpha = \lim \frac{\Delta\ell}{\Delta\tau^2/2} \rightarrow$ $[\alpha] = LT^{-2}$

Relativistic momentum equation: $dp^\mu/d\tau = f^\mu$

For test particle (rest mass m): $f^\mu = mA^\mu$

Thus $|f| = m\alpha$ with $[m] = \mathrm{T}^2\mathrm{L}^{-1}$ (from $m_0 = \hbar\omega_0/c^2$, $[c^2] = \mathrm{L}^2\mathrm{T}^{-2}$)

$[f] = (T^2L^{-1})(LT^{-2}) = 1$

5 Transition from 2019 SI

2019 SI: $\hbar = 1.054\,571\,800 \times 10^{-34}$ J s

Updated SI: replace with $\omega_0 = m_0c^2/\hbar$

Ratio $\hbar = m_0c^2/\omega_0$ becomes exact $\rightarrow$ $m_0$becomes exact $\rightarrow$every other rest mass $m = m_0 \times$ (measured frequency ratio) becomes dimensionless multiple of m_0.